Cosmological model with viscosity media (dark fluid) described by an effective equation of state

Jie Ren[1,*] and Xin-He Meng[1,2,†]

[1]Department of physics, Nankai University, Tianjin 300071, China (post address)
[2]CCAST (World Lab), P.O.Box 8730, Beijing 100080, China

(Dated: February 5, 2008)

A generally parameterized equation of state (EOS) is investigated in the cosmological evolution with bulk viscosity media modelled as dark fluid, which can be regarded as a unification of dark energy and dark matter. Compared with the case of the perfect fluid, this EOS has possessed four additional parameters, which can be interpreted as the case of the non-perfect fluid with time-dependent viscosity or the model with variable cosmological constant. From this general EOS, a completely integrable dynamical equation to the scale factor is obtained with its solution explicitly given out. (i) In this parameterized model of cosmology, for a special choice of the parameters we can explain the late-time accelerating expansion universe in a new view. The early inflation, the median (relatively late time) deceleration, and the recently cosmic acceleration may be unified in a single equation. (ii) A generalized relation of the Hubble parameter scaling with the redshift is obtained for some cosmology interests. (iii) By using the SNe Ia data to fit the effective viscosity model we show that the case of matter described by $p = 0$ plus with effective viscosity contributions can fit the observational gold data in an acceptable level.

PACS numbers: 98.80.Cq, 98.80.-k

I. INTRODUCTION

The cosmological observations have provided increasing evidence that our universe is undergoing a late-time cosmic acceleration expansion [1]. In order to explain the acceleration expansion, cosmologists introduce a new fluid, which possesses a negative enough pressure, called dark energy. According to the observational evidence, especially from the Type Ia Supernovae [2] and WMAP satellite missions[4], we live in a favored spatially flat universe consisting approximately of 30% dark matter and 70% dark energy. The simplest candidate for dark energy is the cosmological constant, but it has got the serious fine-tuning problem. Recently, a great variety of models are proposed to describe the universe with dark energy, partly such as

- Scalar fields: Quintessence [5] and phantom [6], the model potential is from power-law to exponentials and a combination of both.

- Exotic equation of state: Chaplygin gas [7], generalized Chaplygin gas [8], a linear equation of state [9], and Van der Waals equation of state [10].

- Modified gravity: DGP model [12], Cardassian expansion [11], $1/R$, R^2, $\ln R$ term corrections, etc. Maybe the mysterious dark energy does not exist, but we lack the full understanding of gravitational physics [13, 14, 15, 16, 17, 18, 19, 20].

- Viscosity: Bulk viscosity in the isotropic space [22], bulk and shear viscosity in the anisotropic space. The perfect fluid is only an approximation of the universe media. The observations also indicate that the universe media is not a perfect fluid [21] and the viscosity is concerned in the evolution of the universe [23, 24, 25].

We only list a part of the papers on this topics as the relevant are too many. According to Ref. [26],it is possible to put some order in this somewhat chaotic situation by considering a particular feature of the dark energy, namely its equation of state (hereafter EOS); it is tempting to investigate the properties of cosmological models starting from the EOS directly and by testing whether a given EOS is able to give rise to cosmological models reproducing the available dataset. The dark fluid [27] and the parameterized EOS [28] are studied in some recent papers. We hope the situation will be improved with the new generation of more precise observational data.

The observational constraints indicate that the current EOS parameter $w = p/\rho$ is around -1 [2, 3], quite probably below -1, which is called the phantom region and even more mysterious in the cosmological evolution. In the standard model of cosmology, if the $w < -1$, the universe shows to possess the future finite singularity called Big Rip [29, 30]. Several ideas are proposed to prevent the big rip singularity, like by introducing quantum effects terms in the action [32].

Based on the motivations of time-dependent viscosity and modified gravity, the Hubble parameter dependent EOS is considered in Ref. [26, 33], in which the most general "inhomogeneous" EOS is given out, however, they gives analytical solutions of the scale factor only for some less general cases. In this paper, we investigate a general

*Electronic address: jrenphysics@hotmail.com
†Communication author; Electronic address: xhm@nankai.edu.cn

effective equation of state

$$p = (\gamma - 1)\rho + p_0 + w_H H + w_{H2} H^2 + w_{dH} \dot{H},$$

and we show the following time-dependant bulk viscosity

$$\zeta = \zeta_0 + \zeta_1 \frac{\dot{a}}{a} + \zeta_2 \frac{\ddot{a}}{\dot{a}}$$

is equivalent to the form derived by using the above effective EOS. An integrable equation for the scale factor is obtained and three possible interpretations of this equation are proposed. The Hubble parameter dependent term in this EOS can drive the phantom barrier being crossed in an easier way [23, 33, 34]. Different choices of the parameters may lead to several fates to the cosmological evolution [34].

This paper is organized as follows: In the next section we describe our model and give the exact solution of the scale factor. In Sec. III we consider the sound speed and the EOS parameter in this model for unified dark energy, and give some numerical solutions of a more general equation for the scale factor. In Sec. IV we propose three interpretations for our model. In Sec. V we confront the effective viscosity model proposed in the previous Sec. with the SNe Ia Golden data. Finally, we present our conclusions in the last section. The appendix presents some detail comments on the cosmological constant involved.

II. MODEL AND CALCULATIONS

We consider the Friedamnn-Roberson-Walker metric in the flat space geometry ($k = 0$) as favored by WMAP cosmic microwave background data on power spectrum

$$ds^2 = -dt^2 + a(t)^2(dr^2 + r^2 d\Omega^2), \tag{1}$$

and assume that the cosmic fluid possesses a bulk viscosity ζ. The energy-momentum tensor can be written as

$$T_{\mu\nu} = \rho U_\mu U_\nu + (p - \zeta\theta) H_{\mu\nu}, \tag{2}$$

where in comoving coordinates $U^\mu = (1, 0)$, $\theta = U^\mu_{;\mu} = 3\dot{a}/a$, and $H_{\mu\nu} = g_{\mu\nu} + U_\mu U_\nu$ [35]. By defining the effective pressure as $\tilde{p} = p - \zeta\theta$ and from the Einstein equation $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa^2 T_{\mu\nu}$ with $\kappa^2 = 8\pi G$, we obtain the Friedmann equations

$$\frac{\dot{a}^2}{a^2} = \frac{\kappa^2}{3}\rho, \tag{3a}$$

$$\frac{\ddot{a}}{a} = -\frac{\kappa^2}{6}(\rho + 3\tilde{p}). \tag{3b}$$

The conservation equation for energy, $T^{0\nu}_{;\nu}$, yields

$$\dot{\rho} + (\rho + \tilde{p})\theta = 0. \tag{4}$$

To describe completely the global behaviors for our Universe evolution an additional relation, a reasonable EOS is required. A generally parameterized EOS can be written as

$$p = (\gamma - 1)\rho + f(\rho; \alpha_i) + g(H, \dot{H}; \alpha_i) \tag{5}$$

where α_i are parameters that are expected that when $\alpha_i \to 0$, the equation of state approaches to that of the perfect fluid, i.e. $p = w\rho$, where the factorized parameter $w = \gamma - 1$ with γ being another parameter. We consider the following EOS, an explicit form as

$$p = (\gamma - 1)\rho + p_0 + w_H H + w_{H2} H^2 + w_{dH} \dot{H}, \tag{6}$$

where p_0, w_H, w_{H2}, w_{dH} are free parameters. In this and the next section, we assume the universe media is a single fluid described by this EOS. Compared with the bulk viscosity form as described in Ref. [34], the following one is more general. We show that this time-dependent bulk viscosity

$$\zeta = \zeta_0 + \zeta_1 \frac{\dot{a}}{a} + \zeta_2 \frac{\ddot{a}}{\dot{a}} \tag{7}$$

is effectively equivalent to the form derived by using Eq. (6). The reason is

$$\begin{aligned} \tilde{p} &= p - \zeta\theta \\ &= p - 3\zeta_0 \frac{\dot{a}}{a} - 3\zeta_1 \frac{\dot{a}^2}{a^2} - 3\zeta_2 \frac{\ddot{a}}{a} \\ &= p - 3\zeta_0 \frac{\dot{a}}{a} - 3(\zeta_1 + \zeta_2)\frac{\dot{a}^2}{a^2} - 3\zeta_2 \left(\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}\right) \\ &= p - 3\zeta_0 H - 3(\zeta_1 + \zeta_2) H^2 - 3\zeta_2 \dot{H}, \end{aligned} \tag{8}$$

we can see that the corresponding coefficients are

$$w_H = -3\zeta_0, \tag{9a}$$

$$w_{H2} = -3(\zeta_1 + \zeta_2), \tag{9b}$$

$$w_{dH} = -3\zeta_2. \tag{9c}$$

The motivation of considering this bulk viscosity is that by fluid mechanics we know the transport/viscosity phenomenon is related to the "velocity" $\dot{a}$, which is related to the Hubble parameter, and the acceleration. Since we do not know the exact form of viscosity, here we consider a parameterized bulk viscosity, which is a linear combination of three terms: the first term is a constant ζ_0, the second corresponds to the Hubble parameter, and the third can be proportional to $\ddot{a}/aH$. From the above corresponding coefficients, we can see that the inhomogeneous EOS may be interpreted simply as time-dependent viscosity case. Additionally, the EOS of Eq. (6) can also be interpreted as the case of a variable cosmological constant model, in which the Λ-term is written as

$$\Lambda = \Lambda_0 + \Lambda_H H + \Lambda_{H2} H^2 + \Lambda_{dH} \dot{H}. \tag{10}$$

Using this EOS to eliminate ρ and p, we obtain the equation which determines the scale factor $a(t)$ evolution

$$\frac{\ddot{a}}{a}=\frac{-(3\gamma-2)/2-(\kappa^2/2)w_{H2}+(\kappa^2/2)w_{dH}}{1+(\kappa^2/2)w_{dH}}\frac{\dot{a}^2}{a^2}+\frac{-(\kappa^2/2)w_H}{1+(\kappa^2/2)w_{dH}}\frac{\dot{a}}{a}+\frac{-(\kappa^2/2)p_0}{1+(\kappa^2/2)w_{dH}}. \tag{11}$$

To make this equation more comparable to that of the perfect fluid, we define $\tilde{\gamma}$ given by

$$\frac{-(3\gamma-2)/2-(\kappa^2/2)w_{H2}+(\kappa^2/2)w_{dH}}{1+(\kappa^2/2)w_{dH}}=-\frac{3\tilde{\gamma}-2}{2}. \tag{12}$$

This equation gives

$$\tilde{\gamma}=\frac{\gamma+(\kappa^2/3)w_{H2}}{1+(\kappa^2/2)w_{dH}}. \tag{13}$$

By defining

$$\frac{1}{T_1} = \frac{-(\kappa^2/2)w_H}{1+(\kappa^2/2)w_{dH}} \tag{14}$$

$$\frac{1}{T_2^2} = \frac{-(\kappa^2/2)p_0}{1+(\kappa^2/2)w_{dH}}, \tag{15}$$

$$\frac{1}{T^2} = \frac{1}{T_1^2}+\frac{6\tilde{\gamma}}{T_2^2}. \tag{16}$$

and noting that dim[T_1]=dim[T_2]=[time], we can see that when $T_2\to\infty$, $T=T_1$; when $T_1\to\infty$, $T=T_2\sqrt{6\tilde{\gamma}}$. Now Eq. (11) becomes

$$\frac{\ddot{a}}{a}=-\frac{3\tilde{\gamma}-2}{2}\frac{\dot{a}^2}{a^2}+\frac{1}{T_1}\frac{\dot{a}}{a}+\frac{1}{T_2^2}. \tag{17}$$

The five parameters γ, p_0, w_H, w_{H2}, and w_{dH} are condensed to three parameters $\tilde{\gamma}$, T_1, and T_2 in the above equation.

With the initial conditions of $a(t_0)=a_0$ and $\theta(t_0)=\theta_0$, if $\tilde{\gamma}\neq 0$, the solution can be obtained as

$$a(t)=a_0\left\{\frac{1}{2}\left(1+\tilde{\gamma}\theta_0 T-\frac{T}{T_1}\right)\exp\left[\frac{t-t_0}{2}\left(\frac{1}{T}+\frac{1}{T_1}\right)\right]+\frac{1}{2}\left(1-\tilde{\gamma}\theta_0 T+\frac{T}{T_1}\right)\exp\left[-\frac{t-t_0}{2}\left(\frac{1}{T}-\frac{1}{T_1}\right)\right]\right\}^{2/3\tilde{\gamma}}, \tag{18}$$

And we obtain directly

$$\rho(t)=\frac{3}{\kappa^2}\frac{\dot{a}^2}{a^2}=\frac{1}{3\kappa^2\tilde{\gamma}^2}\left[\frac{(1+\tilde{\gamma}\theta_0T-\frac{T}{T_1})(\frac{1}{T}+\frac{1}{T_1})\exp(\frac{t-t_0}{T})-(1-\tilde{\gamma}\theta_0T+\frac{T}{T_1})(\frac{1}{T}-\frac{1}{T_1})}{(1+\tilde{\gamma}\theta_0T-\frac{T}{T_1})\exp(\frac{t-t_0}{T})+(1-\tilde{\gamma}\theta_0T+\frac{T}{T_1})}\right]^2. \tag{19}$$

The above solution is valid when $\tilde{\gamma}\neq 0$. For $\tilde{\gamma}=0$, we need to take the limit case. When $\tilde{\gamma}\to 0$, the limit of the solution $a(t)$ is got as

$$a(t)=a_0\exp\left[\left(\frac{1}{3}\theta_0T_1+\frac{T_1^2}{T_2^2}\right)\left(e^{(t-t_0)/T_1}-1\right)-\frac{T_1(t-t_0)}{T_2^2}\right]. \tag{20}$$

And we obtain directly

$$\rho(t)=\frac{3}{\kappa^2}\left[\frac{1}{3}\theta_0 e^{(t-t_0)/T_1}+\frac{T_1}{T_2^2}\left(e^{(t-t_0)/T_1}-1\right)\right]. \tag{21}$$

Note that the solution $a(t)$ for $\tilde{\gamma}=0$ has not possessed the future singularity, the so called Big Rip, in this case.

III. SOUND SPEED AND EOS PARAMETER

According to Ref. [2], the observational consequences are summarized as follows:

- They provide the first conclusive evidence for cosmic deceleration that preceded the current epoch of cosmic acceleration. Using a simple model of the expansion history, the transition between the two epochs is constrained to be at $z = 0.46 \pm 0.13$.
- For a flat universe with a cosmological constant, they measure $\Omega_M = 0.29\pm^{0.05}_{0.03}$ (equivalently, $\Omega_\Lambda = 0.71$).
- When combined with external flat-universe constraints including the cosmic microwave background and large-scale structure, they find $w = -1.02\pm^{0.13}_{0.19}$ (and $w < -0.76$ at the 95% confidence level) for an assumed static equation of state of dark energy, $p = w\rho$.
- The constraints are consistent with the static nature of and value of w expected for a cosmological constant (i.e., $w_0 = -1.0$, $dw/dz = 0$), and are inconsistent with very rapid evolution of dark energy.

On the basis of the above observational consequences, we suggest that $\tilde{\gamma} \sim 0$ and the parameter T_1 be negative, if the dark fluid describes the unification of dark matter and dark energy. Because when $\tilde{\gamma} = 0$ and $T_1 < 0$, we can obtain

- The universe can accelerate after the epoch of deceleration.
- The density approaches to a constant in the late times, which corresponds to the de Sitter universe, so there is no future singularity.
- The EOS parameter w approaches to -1 when the cosmic time t is sufficiently large.
- The sound speed is a real number (see Ref. [31] for constraints of sound speed).

Because of these features, we construct a model of dark fluid which can be seen as a unification of dark energy and dark matter. Using Eq. (3a), we obtain the relation between p and ρ

$$p = (\gamma - 1)\rho + p_0 + \frac{\kappa}{\sqrt{3}} w_H \sqrt{\rho} + \frac{\kappa^2}{3} w_{H2}\rho^2 - \frac{\kappa^2}{2} w_{dH}(p+\rho), \tag{22}$$

that is

$$(1 + \frac{\kappa^2}{2} w_{dH})p = (\gamma - 1 + \frac{\kappa^2}{3} w_{H2} - \frac{\kappa^2}{2} w_{dH})\rho + \frac{\kappa}{\sqrt{3}} w_H \sqrt{\rho} + p_0. \tag{23}$$

So the EOS between p and ρ is

$$p = (\tilde{\gamma} - 1)\rho - \frac{2}{\sqrt{3}\kappa T_1}\sqrt{\rho} - \frac{2}{\kappa^2 T_2^2}, \tag{24}$$

where $\tilde{\gamma}$ is defined as before. Figs. 1 and 2 show that the parameters γ, w_{H2}, and w_{dH} can drive $\tilde{\gamma}$ crossing -1. In the present paper, we set $\kappa = 1$ and $\theta_0 = 1$ for simplicity to all the figures in this paper. The values of other parameters are given in the legend and the caption of each figure. We especially consider the following choice of the parameters: $\gamma = 0$, $T_1 = -25$, and $T_2 = 100$.

The square of the sound speed is

$$c_s^2 = \frac{\partial p}{\partial \rho} = \tilde{\gamma} - 1 - \frac{1}{\sqrt{3}\kappa T_1}\frac{1}{\sqrt{\rho}}. \tag{25}$$

When $\tilde{\gamma} = 0$, T_1 should be negative if the sound speed is a real number. The graph of c_s^2-t relations is shown in Fig. 3. We can see that the sound speed approaches to a constant in the late times.

The EOS parameter is

$$w = \frac{p}{\rho} = \tilde{\gamma} - 1 - \frac{2}{\sqrt{3}\kappa T_1}\frac{1}{\sqrt{\rho}} - \frac{2}{\kappa^2 T_2^2}\frac{1}{\rho} \tag{26}$$

Fig. 4 shows the w-t relation. This figure shows that w also approaches to a constant in the late times. This is because the density ρ approaches to a constant after some time. Because we have already chosen $\tilde{\gamma} = 0$, there is no $w = -1$ crossing. However, if $\tilde{\gamma}$ is around zero, the crossing may easily occur. Since we chose the parameter T_1 to be negative, from Eq. (19), we can see that the density approches to a constant

$$\rho = -\frac{3T_1}{\kappa^2 T_2^2} \tag{27}$$

after a sufficiently large time, as in Fig. 5.

In order to explain the observations, there should be at least one term in the right hand side of Eq. (17) causing the cosmic expansion to accelerate and one forcing the expansion to decelerate. If we assume that the universe approaches to the de Sitter space-time in the late times, $(+,-,+)$, $(-,+,+)$, and $(-,-,+)$ are possible combinations of the signs of the three terms. It is interesting that Eq. (17) with the signs $(+,-,+)$ in the right hand side may unify the early-time inflation, the middle-time deceleration and the late-time acceleration, which is discussed

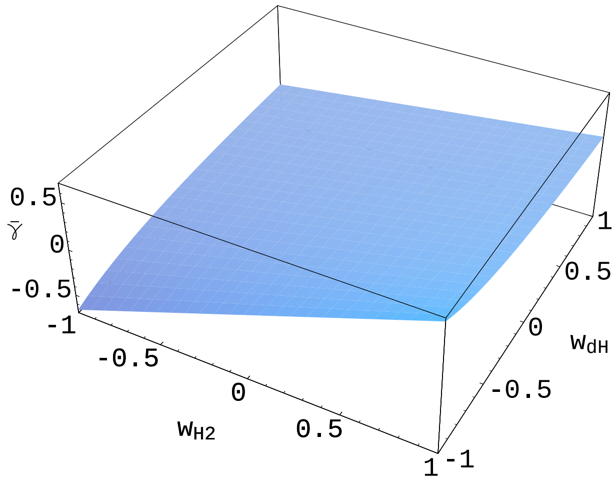

FIG. 1: The relation of $\tilde{\gamma}$, w_{H2}, and w_{dH} when $\gamma = 0$.

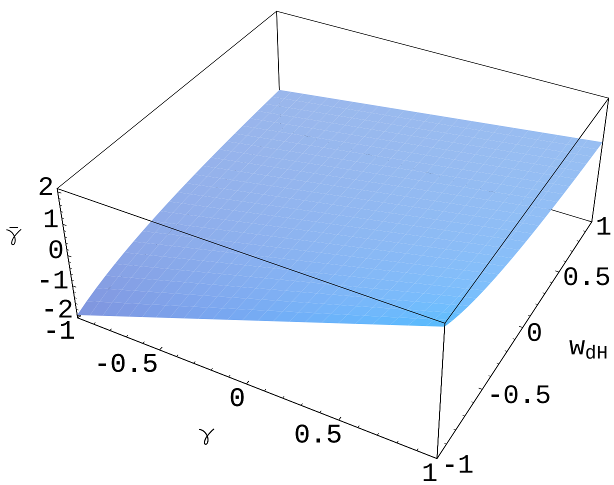

FIG. 2: The relation of $\tilde{\gamma}$, γ, and w_{H2} when $w_{H2} = 0$

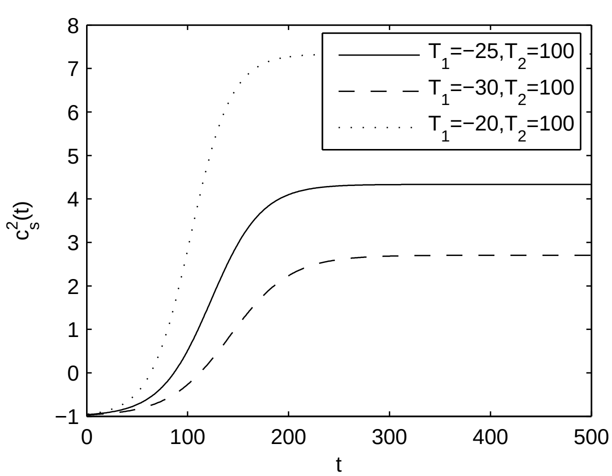

FIG. 3: The relation between the square of the sound speed c_s^2 and the cosmic time t.

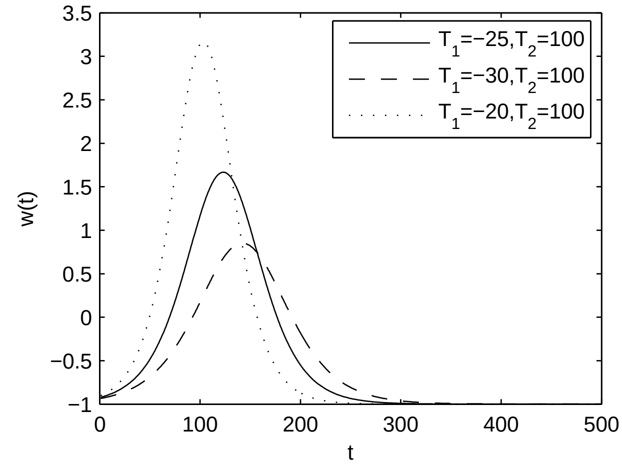

FIG. 4: The relation between the EOS parameter $w = p/\rho$ and the the cosmic time t.

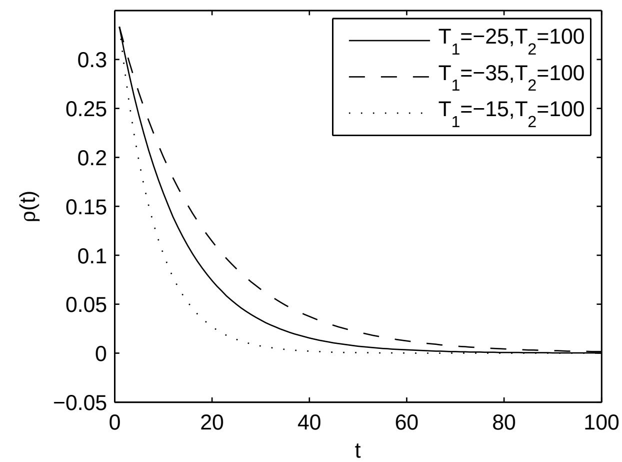

FIG. 5: The relation between the density ρ and the cosmic time. Note that the density approaches to a constant, which is not zero.

in the next section. Fig. 6 shows that the universe accelerates after an epoch of deceleration, and Fig. 7 shows the corresponding evolution of the scale factor. The case for possible future singularity is considered in our previous paper [34]. In Ref. [32, 37, 38], they demonstrate that the quantum effects play the dominant role near/before a big rip, driving the universe out of a future singularity (or at least, moderating it). It is also interesting to study the entropy and dissipation [39, 40, 41], since this EOS may be interpreted as the time-dependent viscosity case.

The more general EOS, such as the form

$$p_X = -\rho_X - A\rho_X^{\alpha} - BH^{2\beta}, \tag{28}$$

in Ref. [26], give more general dynamical equations,

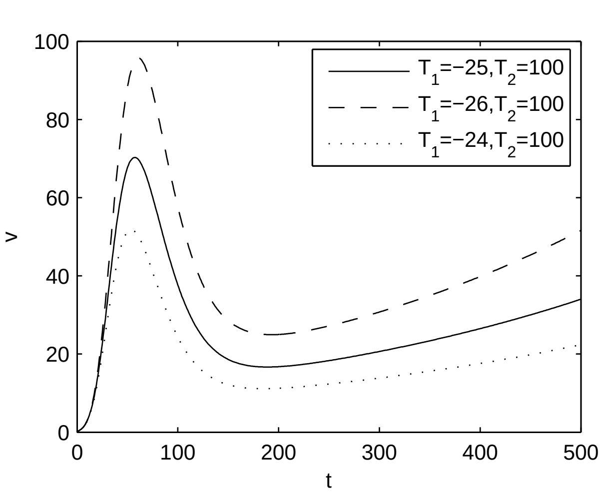

FIG. 6: The relation between the expansion velocity $v = \dot{a}$ and the cosmic time t.

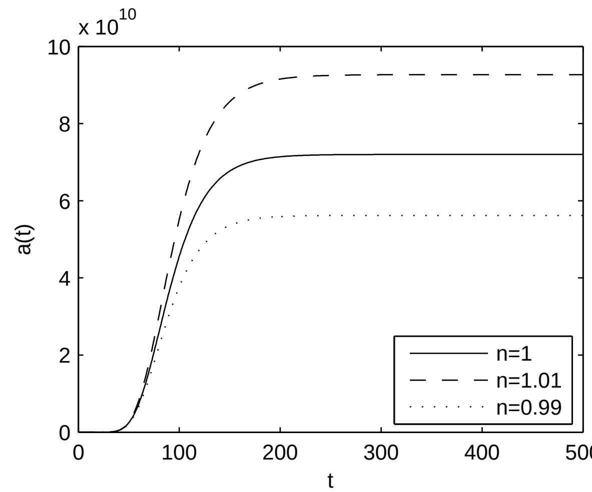

FIG. 8: The evolution of the scale factor when $\gamma = -1$ and $t_c = -25$.

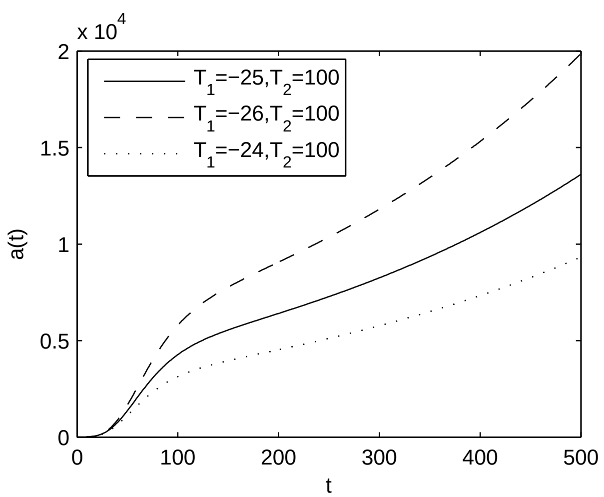

FIG. 7: The relation between the scale factor a and the cosmic time t.

which can be written as

$$\frac{\ddot{a}}{a} = -\frac{3\tilde{\gamma}-2}{2}\frac{\dot{a}^2}{a^2} + \lambda\left(\frac{\dot{a}}{a}\right)^m + \mu\left(\frac{\dot{a}}{a}\right)^n + \nu. \qquad (29)$$

The corresponding coefficients to Eq. (28) are $\tilde{\gamma} = 0$, $\lambda = A(\kappa^2/2)(3/\kappa^2)^\alpha$, $m = 2\alpha$, $\mu = (\kappa^2/2)B$, $n = 2\beta$, and $\nu = 0$. Now we only consider a simpler case to illustrate the scale factor evolution behaviors,

$$\frac{\ddot{a}}{a} = -\frac{3\tilde{\gamma}-2}{2}\frac{\dot{a}^2}{a^2} + \frac{1}{t_c}\left(\frac{\dot{a}}{a}\right)^n. \qquad (30)$$

where t_c is a parameter. Fig. 8 shows the evolution of the scale factor with different n.

IV. INTERPRETATIONS OF THE MODEL

A. Unified dark energy

Since we do not know the nature of either dark energy or dark matter, maybe they can be regarded as two aspects of a single fluid. Based on the analysis in the above section, the EOS in our model can be looked as that of the unified dark energy, since the universe expansion can accelerate after the epoch of deceleration. The model in the present paper can also be regarded as the ΛCDM model with an additional term, or the ΛCDM model with bulk viscosity. The parameter space is enriched in this model. The ΛCDM model describes two mixed fluids, and their EOSs are $p = 0$ for dark matter and $p = -\rho$ for dark energy (cosmological constant). In our model, the case $\tilde{\gamma} = 1$ and $T_1 \to \infty$ corresponds to the ΛCDM model. However, in the above section, we study a special choice of the parameters, $\tilde{\gamma} = 0$, $T_1 = -25$, and $T_2 = 100$, which is totally different from the ΛCDM model.

The qualitative analysis of Eq. (17) can be easily obtained if we assume that H is always decreasing during the cosmic evolution. The three terms in the right hand side of Eq. (17) are proportional to H^2, H^1, and H^0, respectively. If we assume $a \propto e^{2t}$, then $H \propto e^t$, so the proportions of the three terms are $e^{2t} : e^t : 1$; if we assume $a \propto t^{2/3}$, then $H \propto 1/t$, so the proportions of the three terms are $t^{-2} : t^{-1} : 1$. In the early times, the first term is dominant, which may lead to inflation if $\tilde{\gamma} \sim 0$. In the roughly middle times, the second term is dominant, which leads to deceleration if $T_1 < 0$. In the late times as current, the third term is dominant, which leads to acceleration like the de Sitter universe if T_2 is a real number. We can also see the evolution of $\dot{a}(t)$ in Fig. 6 and $a(t)$ in Fig. 7.

In Eq. (17), the term $\frac{1}{T_1}\frac{\dot{a}}{a}$ describes the effective viscosity. Since we do not know much about the nature of dark

energy and the bulk viscosity in the universe, so the bulk viscosity can be regarded as effective, or a contribution as friction term.In order to separately study the effect of the three terms in the right hand side of Eq. (17), if the first, and the second term are dominant, respectively we have the evolution relations

$$\frac{\ddot{a}}{a} = -\frac{3\gamma-2}{2}\frac{\dot{a}^2}{a^2} \;\Rightarrow\; a\frac{dH_\gamma}{da} = -\frac{3\gamma}{2}H_\gamma, \tag{31a}$$

$$\frac{\ddot{a}}{a} = \frac{1}{T_1}\frac{\dot{a}}{a} \;\Rightarrow\; a\frac{dH_v}{da} = -H + \frac{1}{T_1}. \tag{31b}$$

The solutions are correspondingly different

$$H_\gamma(z) = H_0^2(1+z)^{3\gamma}, \tag{32a}$$

$$H_v(z) = \left(H_0 - \frac{1}{T_1}\right)(z+1) + \frac{1}{T_1}. \tag{32b}$$

B. Mixture of dark energy and dark matter

Another interpretation is that the EOS describes the dark energy, which is mixed with the dark matter in the universe media. So we should concern on the mixture of the dark energy and dark matter, which requires fine-tuning of the parameters. In the ΛCDM model of cosmology, we have

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2[\Omega_m(1+z)^3 + \Omega_\Lambda]. \tag{33}$$

where H_0 is the current value of the Hubble parameter, $z = a_0/a - 1$ is the redshift, Ω_m and Ω_Λ are the cosmological density parameters of matter and Λ-term, respectively. In our case,

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2\Omega_m(1+z)^3 + (1-\Omega_m)H_d(z)^2, \tag{34}$$

where $H_d(z)$ is the solution of the equation

$$aH\frac{dH}{da} = -\frac{3\tilde{\gamma}}{2}H^2 + \frac{1}{T_1}H + \frac{1}{T_2^2}. \tag{35}$$

The solution of the above equation with the initial condition $H(a_0) = H_0$ is

$$\left|\frac{(H - \frac{1}{3\tilde{\gamma}T_1})^2 - \frac{1}{9\tilde{\gamma}^2T_1^2} - \frac{2}{3\tilde{\gamma}^2T_2^2}}{(H_0 - \frac{1}{3\tilde{\gamma}T_1})^2 - \frac{1}{9\tilde{\gamma}^2T_1^2} - \frac{2}{3\tilde{\gamma}^2T_2^2}}\right| = (1+z)^{3\gamma}. \tag{36}$$

Here we consider a simpler case, $T_2 \to \infty$, then

$$a\frac{dH}{da} = -\frac{3\tilde{\gamma}}{2}H + \frac{1}{T_1}. \tag{37}$$

The solution is

$$H(a) = \left(H_0 - \frac{2}{3\tilde{\gamma}T_1}\right)\left(\frac{a}{a_0}\right)^{-3\gamma/2} + \frac{2}{3\tilde{\gamma}T_1}, \tag{38}$$

so $H(z)$ for the dark energy is

$$H_d(z) = \left(H_0 - \frac{2}{3\tilde{\gamma}T_1}\right)(z+1)^{3\gamma/2} + \frac{2}{3\tilde{\gamma}T_1}. \tag{39}$$

It is interesting that the above relation can be rewritten as

$$H_d(z) = H_0[\tilde{\Omega}(1+z)^{3\tilde{\gamma}/2} + (1-\tilde{\Omega})], \tag{40}$$

where

$$\tilde{\Omega} = 1 - \frac{2}{3\tilde{\gamma}T_1H_0}. \tag{41}$$

Note that Eq. (40) is valid if $\tilde{\gamma} \neq 0$, and for the case $\tilde{\gamma} = 0$, directly solving Eq. (37) gives

$$H_d(z) = H_0\left[1 - \frac{1}{T_1H_0}\ln(z+1)\right]. \tag{42}$$

We assume the universe media contains two fluids. One is described by Eq. (17) with $T_2 \to \infty$, and another is described by the simplest pure cosmological constant Λ. The former may be regarded as the dark matter with effective viscosity. The H-z relation is thus

$$H^2 = H_0^2\{\Omega_m[\tilde{\Omega}(1+z)^{3\tilde{\gamma}/2} + (1-\tilde{\Omega})]^2 + (1-\Omega_m)\}, \tag{43}$$

which can be regarded as the generalized relation of mixed dark energy and dard matter. We emphasize that solving Eq. (35) with $T_2 \to \infty$ and writting $H^2 = \Omega_m H_d^2 + (1-\Omega_m)H_0^2$ is not equivalent to directly solving Eq. (35) (see Appendix for details).

If the universe media can be perceived as the mixture of matter, radiation, Λ term, and with effective viscosity, by ignoring curvature contribution as favored from WMAP data. The total density is

$$\rho = \rho_m + \rho_r + \rho_\Lambda + \rho_v, \tag{44}$$

where the subscripts denote the matter, radiation, Λ, and effective viscosity components, respectively. Since $\rho \propto H^2$, we have

$$H(z)^2 = \Omega_m H_m^2 + \Omega_r H_r^2 + \Omega_\Lambda H_\Lambda^2 + \Omega_v H_v^2. \tag{45}$$

C. Effective viscosity model

The third interpretation is a new model called effective viscosity model, which may be a most significant result in this paper. We assume the universe media can be described by only a single fluid, which corresponds to the matter described by the EOS of $p = 0$, and with an effective constant viscosity. In this model, it is the effective viscosity that causes the cosmic expansion acceleration without by introducing a cosmological constant, which is totally different from the ΛCDM model. We rewrite Eq. (40) with $\tilde{\gamma} = 1$ as

$$H(z) = H_0[\Omega_m(1+z)^{3/2} + (1-\Omega_m)] \tag{46}$$

There is one adjustable parameter in this model. In the next section we will show that this model can fit the SNe Ia data at an acceptable level.

V. DATA FITTING OF THE EFFECTIVE VISCOSITY MODEL

The observations of the SNe Ia have provided the first direct evidence of the accelerating expansion for our current universe. Any model attempting to explain the acceleration mechanism should be consistent with the SNe Ia data implying results, as a basic requirement. The method of the data fitting is illustrated in Ref. [36]. The observations of supernovae measure essentially the apparent magnitude m, which is related to the luminosity distance d_L by

$$m(z) = \mathcal{M} + 5\log_{10} D_L(z), \tag{47}$$

where $D_L(z) \equiv (H_0/c) d_L(z)$ is the dimensionless luminosity distance and

$$d_L(z) = (1+z) d_M(z), \tag{48}$$

where $d_M(z)$ is the comoving distance given by

$$d_M(z) = c \int_0^z \frac{1}{H(z')} dz'. \tag{49}$$

Also,

$$\mathcal{M} = M + 5\log_{10}\left(\frac{c/H_0}{1\mathrm{Mpc}}\right) + 25, \tag{50}$$

where M is the absolute magnitude which is believed to be constant for all supernovae of type Ia. We use the 157 golden sample of supernovae data compiled by Riess *et al.* [2] to fit our model. The data points in these samples are given in terms of the distance modulus

$$\mu_{obs}(z) \equiv m(z) - M_{obs}(z). \tag{51}$$

The χ^2 is calculated from

$$\chi^2 = \sum_{i=1}^{n} \left[\frac{\mu_{obs}(z_i) - \mathcal{M}' - 5\log_{10} D_{Lth}(z_i; c_\alpha)}{\sigma_{obs}(z_i)} \right]^2. \tag{52}$$

where $\mathcal{M}' = \mathcal{M} - M_{obs}$ is a free parameter and $D_{Lth}(z_i; c_\alpha)$ is the theoretical prediction for the dimensionless luminosity distance of a supernovae at a particular distance, for a given model with parameters c_α. We consider the generalized ΛCDM model as referred to in the previous section and perform a best-fit analysis with the minimization of the χ^2, with respect to $\mathcal{M}'$, Ω_m. Fig. 9 shows that the theoretical curve fits the observational data at an acceptable level, with only one adjustable parameter Ω_m except the today's Hubble parameter H_0.

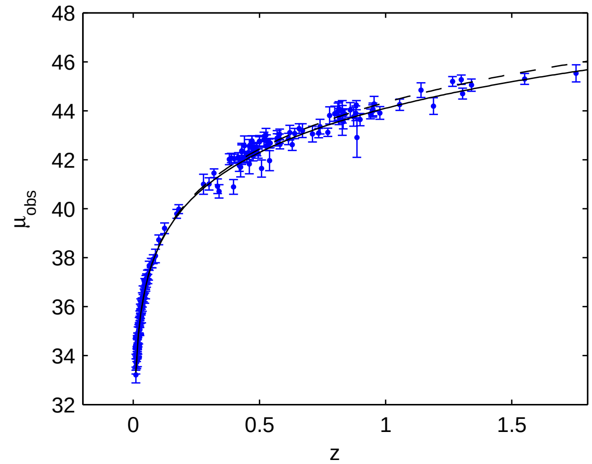

FIG. 9: The dependence of luminosity on redshift computed from the effective viscosity model. The solid and dashed lines correspond to $\Omega_m = 0.3$ and $\Omega_m = 0.5$, respectively. The dots are the observed data.

VI. DISCUSSION AND CONCLUSION

We investigate a parameterized effective EOS of dark fluid in the cosmological evolution. With this general EOS, the dynamical equation of the scale factor is completely integrable and an exact solution for Einstein's gravitational equation with FRW metric is obtained. The parameters γ, p_0, w_H, w_{H2}, and w_{dH} can be reduced to three condensed parameters $\tilde{\gamma}$, T_1, and T_2. Three interpretations to this model are proposed in this paper:

- This EOS can be regarded as a unification of the dark energy and dark matter, so there is a single fluid to show functions in the universe. In this case, we prefer to the choice of the parameters: $\tilde{\gamma} \sim 0$, $T_1 < 0$, and $T_2^2 > 0$.

- This EOS describes the dark energy, which is mixed with the dark matter in the universe media; or this EOS describes the dark matter with viscosity, which is mixed with the dark energy from the Λ-term.

- The universe media contains a single fluid, which corresponds to the matter described by the EOS of $p = 0$, with an effectively constant viscosity. It is the effective viscosity that causes the cosmic expansion acceleration without by introducing a cosmological constant. In this case, we prefer to the choice of the parameters: $\tilde{\gamma} \sim 1$, $T_1 > 0$, and $T_2 = 0$.

Different choices of the parameters may lead to several fates of the cosmological evolution. We especially study the choices for the parameters of $\tilde{\gamma} = 0$ and $T_1 < 0$ and the unified dark energy in the first interpretation case. We presents a generalized relation of H-z compared with the ΛCDM model. We show that the matter described by the EOS of $p = 0$ plus with effective viscosity and without introducing the cosmological constant can fit the observational data well, so the effective viscosity model

may be an alternative candidate to explain the late-time accelerating expansion universe.

APPENDIX A: REMARKS ON THE Λ-TERM INVOVLED

Directly solving Eq. (35) with the EOS $p = (\gamma - 1)\rho$ gives

$$H(z) = \left(H_0^2 - \frac{2}{3\tilde{\gamma}^2 T_2^2}\right)(1+z)^{3\gamma} + \frac{2}{3\tilde{\gamma}^2 T_2^2}, \quad \text{(A1)}$$

which can be rewritten as

$$H^2 = H_0^2[\Omega_m(1+z)^{3\gamma} + (1-\Omega_m)], \quad \text{(A2)}$$

where $\Omega_m = 1 - \frac{2}{3\tilde{\gamma}^2 T_2^2 H_0^2}$. Solving the Friedmann equations with the EOS $p = (\gamma-1)\rho$ without the Λ-term gives $H_x^2 = H_0^2(1+z)^{3\gamma}$. On the other hand, concerning on the mixture of the dark energy and dark matter, we can write $H^2 = \Omega_m H_x^2 + (1-\Omega_m)H_0^2$, which is exactly the same as Eq. (A2). However, the following two methods are not equivalent except for some very special cases: (i) Using the EOS $p = f(\rho)$ to solve the Friedmann equations with the Λ-term, we obtain the H-z relation. (ii) Using the EOS $p = f(\rho)$ to solve the Friedmann equations without the Λ-term, we obtain $H_x(z)$ and write

$$H^2 = \Omega_m H_x(z)^2 + (1-\Omega_m)H_0^2. \quad \text{(A3)}$$

Generally the above H-z relation is not equivalent to what is obtained in (i).

ACKNOWLEDGEMENTS

X.H.M. is very grateful to Profs. S.D. Odintsov and I. Brevik for lots of helpful comments with reading the main contents. This work is supported partly by NSF and Doctoral Foundation of China.

[1] T. Totani, Y. Yoshii, and K. Sato, Astrophys. J. **483**, L75 (1997); S. Perlmutter *et al.*, Nature **391**, 51 (1998); A.G. Riess *et al.*, Astron. J. **116**, 1009 (1998); N. Bahcall, J.P. Ostriker, S. Perlmutter, and P.J. Steinhardt, Science **284**, 1481 (1999).

[2] A.G. Riess *et al.*, Astrophys. J. **607**, 665 (2004);

[3] H.Jassal, J.Bagla and T.Padmanabhan, Mon.Not.Roy.Astron.Soc.Letters 356, L11 (2005); astro-ph/0506748

[4] C. L. Bennett *et al.*, Astrophys. J. Suppl. **148**, 1 (2003).

[5] L. Wang, R.R. Caldwell, J.P. Ostriker, and P.J. Steinhardt, Astrophys. J. **530**, 17 (2000).

[6] R.R. Caldwell, Phys. Lett. B **545**, 23 (2002), astro-ph/9908168.

[7] A. Kamenshchik, U. Moschella, and V. Pasquier, Phys. Lett. B **511**, 265 (2001).

[8] M.C. Bento, O. Bertolami, and A.A. Sen, Phys. Rev. D **66**, 043507 (2002).

[9] E. Babichev, V. Dokuchaev, and Y. Eroshenko, Class. Quantum Grav. **22**, 143 (2005).

[10] S. Capozziello, S.D. Martino, and M. Falanga, Phys. Lett. B **299**, 494 (2002).

[11] K. Freese and M. Lewis, Phys. Lett. B **540**, 1 (2002).

[12] G.R. Dvali, G. Gabadadze, and M. Porrati, Phys. Lett. B **484**, 112 (2000); G.R. Dvali and M.S. Turner, astro-ph/0301510.

[13] A. Lue, R. Scoccimarro, and G. Starkman, Phys. Rev. D **69**, 044005 (2004).

[14] S. Nojiri, S.D. Odintsov, Phys. Lett. B **599**, 137 (2004).

[15] S.M. Carroll, V. Duvvuri, M. Trodden, and M. S. Turner, Phys. Rev. D **70**, 043528 (2004).

[16] X.H. Meng and P. Wang, Class. Quant. Grav. **20**, 4949 (2003); ibid, **21**, 951 (2004); ibid, **22**, 23 (2005); Phys. Lett. B **584**, 1 (2004).

[17] T. Chiba, Phys. Lett. B **575**, 1, (2003), astro-ph/0307338.

[18] E.E. Flanagan, Phys. Rev. Lett. 92, 071101, (2004), astro-ph/0308111.

[19] S. Nojiri and S.D. Odintsov, Phys. Rev. D **68**, 123512 (2003), hep-th/0307288.

[20] D.N. Vollick, Phys. Rev. D **68**, 063510, (2003), astro-ph/0306630.

[21] T.R. Jaffe, A.J. Banday, H.K. Eriksen, K.M. Górski, and F.K. Hansen, astro-ph/0503213.

[22] T.Padmanabhan and S.Chitre, Phys.Lett.A120, 433 (1987)

[23] I. Brevik and O. Gorbunova, gr-qc/0504001.

[24] I. Brevik, O. Gorbunova, and Y. A. Shaido, gr-qc/0508038.

[25] M. Cataldo, N. Cruz, and S. Lepe, Phys. Lett. B **619**, 5 (2005).

[26] S. Capozziello, V.F. Cardone, E. Elizalde, S. Nojiri, and S.D. Odinsov, astro-ph/0508350.

[27] A. Arbey, astro-ph/0506732; astro-ph/0509592; X.H. Meng, M.G. Hu, and J. Ren, astro-ph/0510357.

[28] V.B. Johri and P.K. Rath, astro-ph/0510017.

[29] R.R. Caldwell, M. Kamionkowski, and N.N. Weinberg, Phys. Rev. Lett. **91**, 071301 (2003).

[30] S. Nojiri, S.D. Odintsov, and S. Tsujikawa, Phys. Rev. D **71**, 063004 (2005).

[31] S. Hannestad, Phys. Rev. D **71**, 103519 (2005).

[32] S. Nojiri and S.D. Odintsov, Phys. Lett. B **595**, 1 (2004), hep-th/0405078.

[33] S. Nojiri and S.D. Odintsov, Phys. Rev. D **72**, 023003 (2005).

[34] X.H. Meng, J. Ren, and M.G. Hu, astro-ph/0509250.

[35] I. Brevik, Phys. Rev. D **65**, 127302 (2002).

[36] M.C. Bento, O. Bertolami, N.M.C. Santos, and A.A. Sen, Phys. Rev. D **71**, 063501 (2005).

[37] S. Nojiri and S.D. Odintsov, Phys. Rev. D **70**, 103522 (2004).

[38] E. Elizalde, S. Nojiri, and S.D. Odintsov, Phys. Rev. D **70**, 043539 (2004), hep-th/0405034.

[39] I. Brevik, S. Nojiri, S.D. Odintsov, and L. Vanzo, Phys.

Rev. D **70**, 043520 (2004).
[40] A.D. Prisco, L. Herrera, and J. Ibanez, Phys. Rev. D **63**, 023501 (2000).
[41] L. Herrera, A.D. Prisco, and J. Ibanez, Class. Quantum Grav. **18**, 1475 (2001).